

Exercise 31

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = x^4$$

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - x^4}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \\ &= \lim_{h \rightarrow 0} (4x^3 + 6x^2h + 4xh^2 + h^3) \\ &= 4x^3 \end{aligned}$$

The domain of $f(x)$ is $\{x \mid -\infty < x < \infty\}$, and the domain of $f'(x)$ is $\{x \mid -\infty < x < \infty\}$. $f(x)$ and $f'(x)$ are polynomials, so any number can be plugged into them.